

Determination of $f_0 - \sigma$ mixing angle through

$B_s^0 \rightarrow J/\Psi f_0(980)(\sigma)$ decays

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Abstract

We study $B_s^0 \rightarrow J/\psi f_0(980)$ decays, the quark content of $f_0(980)$ and the mixing angle of $f_0(980)$ and $\sigma(600)$. We calculate not only the factorizable contribution in QCD factorization scheme but also the nonfactorizable hard spectator corrections in QCDF and pQCD approach. We get consistent result with the experimental data of $B_s^0 \rightarrow J/\psi f_0(980)$ and predict the branching ratio of $B_s^0 \rightarrow J/\psi \sigma$. We suggest two ways to determine $f_0 - \sigma$ mixing angle θ . Using the experimental measured branching ratio of $B_s^0 \rightarrow J/\psi f_0(980)$, we can get the $f_0 - \sigma$ mixing angle θ with some theoretical uncertainties. We suggest another way to determine $f_0 - \sigma$ mixing angle θ using both of experimental measured decay branching ratios $B_s^0 \rightarrow J/\psi f_0(980)(\sigma)$ to avoid theoretical uncertainties.

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I. INTRODUCTION

Scalar mesons are important for testing QCD and the Standard Model(SM). Many scalar mesons have been observed: isoscalar states $\sigma(600)$, $f_0(980)$, $f_0(1500)$, $f_0(1370)$, $f_0(1710)$; the isovector states $a_0(980)$, $a_0(1450)$ and isodoublets $\kappa(800)$, $K_0^*(1430)$ [1]. The number of these scalar mesons exceeds the particle states which can be accommodated in one nonet in the quark model. It is commonly believed that there are two nonets below and above 1GeV [2]-[7]. The meson states in each nonet have not been completely determined yet. Especially, the structure of $f_0(980)$ (abbreviated as f_0) is not settled. The underlying structure of $f_0(980)$ concerns the extraction of the CP -violating phase β_s in $B_s^0 - \bar{B}_s^0$ mixing, defined as $\beta_s = \text{Arg} \left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right]$, which is particularly important for look for new physics(NP)[8]-[13]. The CP -violating phase β_s is predicted to be tiny in the SM: $\beta_s \simeq 0.019$ rad. This is about 20 times smaller in magnitude than the measured value of the corresponding phase 2β in $B^0 - \bar{B}^0$ mixing. Being small, this phase can be drastically increased by the presence of new physics beyond the SM. Thanks to the suppression of light-quark loops, β_s is dominated by short-distance processes and sensitive to NP. Thus, measuring β_s is an important probe for new physics.

Attempts to determine β_s have been made by the CDF, D0, LHCb and ATLAS Collaboration based on the angular analysis of $B_s \rightarrow J/\psi\phi$ [14–17]. Reliable signal of new physics is not founded based on the measured datas of β_s , because of sizable uncertainties due to the strong phases involved in the angular analysis of $B_s \rightarrow J/\psi\phi$ [18]. So the precise measurement of β_s is one of the priorities in the physics programs at the hadron colliders and at the B factories [13, 19]. In Ref. [8] it is argued that in the case of $J/\psi\phi$ final state the analysis is complicated by the presence of an S-wave K^+K^- system interfering with the ϕ . So it is necessary to consider other process to access mixing parameter β_s .

$B_s^0 \rightarrow J/\psi f_0(980)$, which has been observed by the LHCb, CDF and Belle Collaboration recently[9–11], is another promising channel for accessing the mixing parameter. The advantage of this channel is clear: no angular analysis is required because of the $J^P = 0^+$ quantum numbers of the $f_0(980)$. To determine the phase β_s through $B_s^0 \rightarrow J/\psi f_0(980)$, it is essential to study the structure of $f_0(980)$.

The structure of $f_0(980)$ have been investigated in many works[20]–[23]. Studies show that $f_0(980)$ is not a pure $s\bar{s}$ state. The First experimental evidence is the observation of

$\Gamma(J/\psi \rightarrow f_0\omega) \approx \frac{1}{2}\Gamma(J/\psi \rightarrow f_0\phi)$. This result clearly indicates the existence of both the non-strange and strange quark content in $f_0(980)$. The Second evidence is that $f_0(980)$ and $a_0(980)$ have similar widths and that the f_0 width is dominated by $\pi\pi$, that means the existence of $u\bar{u}$ and $d\bar{d}$ pairs in $f_0(980)$. So, $f_0(980) \rightarrow \pi\pi$ should not be OZI suppressed relative to $a_0(980) \rightarrow \pi\eta$. Therefore, isoscalars $\sigma(600)$ and f_0 must have mixing[22],

$$|f_0(980)\rangle = |s\bar{s}\rangle \cos\theta + |n\bar{n}\rangle \sin\theta, \quad |\sigma(600)\rangle = -|s\bar{s}\rangle \sin\theta + |n\bar{n}\rangle \cos\theta, \quad (1)$$

with $n\bar{n} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ and θ is f_0 - σ mixing angle.

Many attempts have been made to determine the f_0 - σ mixing angle. Analysis of experimental data shows that the f_0 - σ mixing angle θ lies in the ranges of $25^\circ < \theta < 40^\circ$ and $140^\circ < \theta < 165^\circ$ [20]-[21]. The f_0 - σ mixing angle is generally determined through the calculation of branching ratios of some mesons decays. In the calculation of the mesons decay amplitudes, some parameters have to be taken as inputs, so the determination of f_0 - σ mixing angle has many uncertainty sources, such as, decay constant, transition form factors, hadron coupling constants, wave functions of the relevant mesons, and assumptions about the variation of the form factors with momentum transfer Q^2 . It is not a good way to determine f_0 - σ mixing angle with too many parameters and assumptions. To extract β_s with better accuracy, it is necessary to find a better method to determine it with less input parameters.

Based on only one conventional assumption that the decay constant and the distribution amplitude of the $s\bar{s}$ component for f_0 is the same as that for σ as in Ref. ([24]-[26]), we can derive the relation between the branching ratios of $B_s^0 \rightarrow J/\Psi f_0(\sigma)$ in Eq. (39). From this relation, we can determine f_0 - σ mixing angle. The only input we need is the experimental value of the ratio of the branching ratios for $B_s^0 \rightarrow J/\Psi f_0(\sigma)$. That means that the f_0 - σ mixing angle determined in this way has much less uncertainty sources.

This paper is organized as follows. In Sec. 2, we derive the formulas for the amplitudes of the $B_s^0 \rightarrow J/\Psi f_0(\sigma)$. Two methods for determining the f_0 - σ mixing angle are presented. Section 3 is for summary and discussion. Some input parameters and mesons wave function are listed in the Appendix.

II. BRANCHING RATIOS FOR THE DECAYS OF $B_s^0 \rightarrow J/\psi f_0(980)$

For the $B_s^0 \rightarrow J/\psi f_0(980)$ decays, the effective Hamiltonian is given by [27],

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}^* V_{cs} [C_1(\mu) O_1 + C_2(\mu) O_2] - V_{tb}^* V_{ts} \sum_{k=3}^{10} C_k(\mu) O_k \right\}, \quad (2)$$

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V and the four-fermion operators,

$$\begin{aligned} O_1 &= (\bar{b}_i c_j)_{V-A} (\bar{c}_j s_i)_{V-A}, & O_2 &= (\bar{b}_i c_i)_{V-A} (\bar{c}_j s_j)_{V-A}, \\ O_3 &= (\bar{b}_i s_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, & O_4 &= (\bar{b}_i s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\ O_5 &= (\bar{b}_i s_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, & O_6 &= (\bar{b}_i s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\ O_7 &= \frac{3}{2} (\bar{b}_i s_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A}, & O_8 &= \frac{3}{2} (\bar{b}_i s_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A}, \\ O_9 &= \frac{3}{2} (\bar{b}_i s_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A}, & O_{10} &= \frac{3}{2} (\bar{b}_i s_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A}, \end{aligned} \quad (3)$$

i, j being the color indices.

In this paper, we take the light-cone coordinates (p^+, p^-, \mathbf{p}_T) to describe the four-dimensional momenta of the meson,

$$p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3), \quad \text{and} \quad \mathbf{p}_T = (p^1, p^2). \quad (4)$$

At the rest frame of the B_s^0 meson, the momentum P_1 of the B_s^0 meson is

$$P_1 = \frac{M_{B_s}}{\sqrt{2}}(1, 1, \mathbf{0}_T) \quad (5)$$

the $J/\Psi(f_0)$ meson momentum $P_2(P_3)$ can be written as

$$P_2 = \frac{M_{B_s}}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T), \quad P_3 = \frac{M_{B_s}}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T) \quad (6)$$

with $r_2 = m_{J/\psi}/M_{B_s}$, $r_3 = m_{f_0}/M_{B_s}$.

The polarization vectors of the J/f_0 meson are parameterized as

$$\epsilon_{2L} = \frac{1}{\sqrt{2}r_2} (1, -r_2^2, \mathbf{0}_T), \quad \epsilon_{2T} = (0, 0, \mathbf{1}_T). \quad (7)$$

The decay width of $B_s^0 \rightarrow J/\psi f_0(980)$ is

$$\Gamma = \frac{1}{32\pi M_{B_s}} G_F^2 (1 - r_2^2 + \frac{1}{2} r_2^4 - r_3^2) |\mathcal{A}|^2. \quad (8)$$

The amplitude \mathcal{A} consists of factorizable part and nonfactorizable part. It can be written as

$$\mathcal{A} = A_{FA} + A_{VERT} + A_{HS}, \quad (9)$$

where A_{FA} denotes the factorizable contribution, A_{VERT} is the vertex corrections from Fig. 1.(a)-(d), A_{HS} is the hard spectator scattering correction from Fig. 1.(e)-(f).

A. Factorizable Contribution and Vertex Correction In QCDF

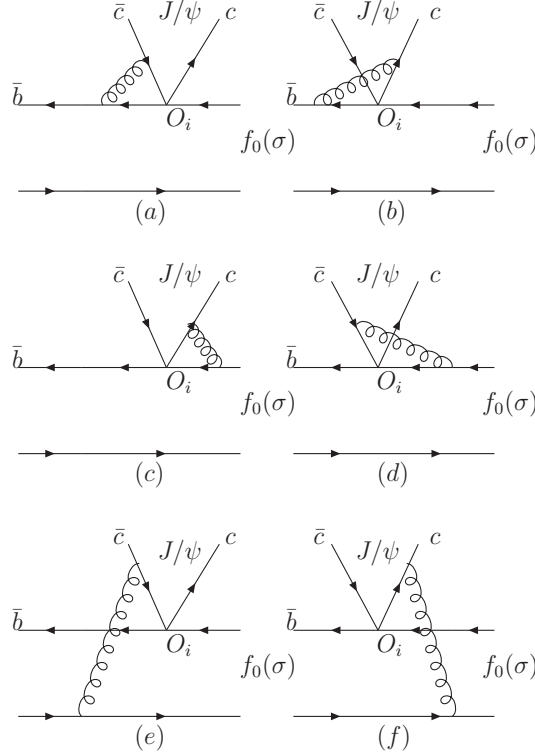


FIG. 1: Nonfactorizable contribution to the $B_s^0 \rightarrow J/\Psi f_0(\sigma)$ decays

The factorizable part A_{FA} of amplitude \mathcal{A} in Eq. (9) for $B_s \rightarrow J/\psi f_0(980)$ decay can not be calculated reliably in pQCD approach, because its characteristic scale is around 1 GeV [28]. We here compute the factorizable part of amplitude and the vertex correction from Fig. 1.(a)-(d) in QCDF [29] instead of pQCD approach and get

$$A_{FA} + A_{VERT} = a_{eff} m_{B_s}^2 \cos \theta f_{J/\psi} F_1^{B_s \rightarrow f_0}(m_{J/\psi}^2)(1 - r_2^2), \quad (10)$$

where $f_{J/\psi}$ is decay constant of J/ψ meson, $F_1^{B_s \rightarrow f_0}$ is the $B_s \rightarrow f_0$ transition form factor defined as

$$\begin{aligned} \langle f_0(P_3) | \bar{b} \gamma_\mu \gamma_5 s | B_s(P_1) \rangle = \\ -i \left\{ F_1^{B_s \rightarrow f_0}(q^2) [(P_1 + P_3)_\mu - \frac{m_{B_s}^2 - m_{f_0}^2}{q^2} q_\mu] + F_0^{B_s \rightarrow f_0}(q^2) \frac{m_{B_s}^2 - m_{f_0}^2}{q^2} q_\mu \right\}, \end{aligned} \quad (11)$$

$q = P_1 - P_3$ being the momentum transfer, and m_{f_0} the f_0 meson mass.

The Wilson coefficient a_{eff} for $B_s \rightarrow J/\psi f_0(980)$ can be derived in QCDF[30, 31],

$$\begin{aligned} a_{eff} = V_c^* \left[C_1 + \frac{C_2}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I \right) \right] \\ - V_t^* \left[C_3 + \frac{C_4}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_4 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I \right) \right] \\ + C_5 + \frac{C_6}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_6 \left(6 - 12 \ln \frac{m_b}{\mu} - f_I \right) \\ + C_7 + \frac{C_8}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_8 \left(6 - 12 \ln \frac{m_b}{\mu} - f_I \right) \\ + C_9 + \frac{C_{10}}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_{10} \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I \right) \Big], \end{aligned} \quad (12)$$

with the function,

$$f_I = \frac{2\sqrt{2N_c}}{f_{J/\psi}} \int dx_2 \Psi^L(x_2) \left[\frac{3(1-2x_2)}{1-x_2} \ln x_2 - 3\pi i + 3 \ln(1-r_2^2) + \frac{2r_2^2(1-x_2)}{1-r_2^2 x_2} \right], \quad (13)$$

and $V_c^* = V_{cb}^* V_{cs}$, $V_t^* = V_{tb}^* V_{ts}$.

For the $B_s \rightarrow f_0$ transition form factor, we employ the models derived from the light-cone sum rules [13], which is parameterized as

$$F_1^{B_s \rightarrow f_0}(q^2) = \frac{F_1^{B_s \rightarrow f_0}(0)}{1 - a_1 q^2 / m_{B_s}^2 + b_1 q^4 / m_{B_s}^4} \quad (14)$$

with $F_1^{B_s \rightarrow f_0}(0) = 0.238$, $a_1 = 1.5$, $b_1 = 0.58$, for $B_s \rightarrow f_0$ transition.

B. Hard Spectator Scattering Corrections In QCDF Approach

For the contribution A_{HS} from hard spectator scattering corrections in Fig. 1.(e)-(f), we can use QCD factorization and get,

$$A_{HS} = \cos \theta f_{J/\psi} \frac{C_F \alpha_s \pi}{N_c^2} (V_c^* C_2 + V_t^* (2C_6 + 2C_8 - C_4 - C_{10})) H_1(M_1 M_2),$$

where $H_1(M_1 M_2)$ is the hard spectator function,

$$H_1(M_1 M_2) = -f_{B_s} \bar{f}_{f_0} \int_0^1 \frac{d\rho}{\rho} \Phi_{B_s}(\rho) \int_0^1 \frac{d\xi}{\xi} \Phi_{J/\psi}(\xi) \int_0^1 \frac{d\eta}{\bar{\eta}} \left[-\Phi_{f_0}(\eta) + r_\chi^{f_0} \frac{\bar{\xi}}{\xi} \Phi_{f_0}^s(\eta) \right], \quad (15)$$

Because twist-3 distribution amplitude $\Phi_{f_0}^s$ of f_0 meson is $\Phi_{f_0}^s = \bar{f}_{f_0}$ [32], the integral $\int_0^1 \frac{d\eta}{\bar{\eta}} \Phi_{f_0}^s(\eta)$ will generate logarithmical divergence from end-point. It is often parameterized as[30],

$$\int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} = \ln\left(\frac{m_{B_s}}{\Lambda_{QCD}}\right) + r \exp(i\delta) \quad (16)$$

where parameter r is often taken from 0 to 6, δ is phase, $0 \leq \delta \leq 2\pi$.

According to Eq. (8)-(16), taking the parameter r varying from 0 to 6, $0 \leq \delta \leq 2\pi$ and other parameters listed in the Appendix, we can get the branching ratio of $B_s \rightarrow J/\psi f_0(980)$. The range of predicted the branching ratio of $B_s \rightarrow J/\psi f_0(980)$ is,

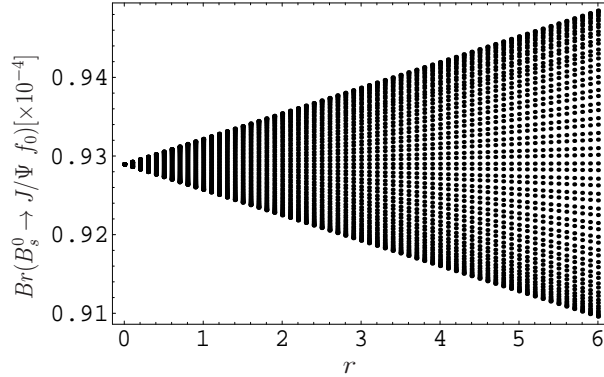


FIG. 2: The range of the branching ratios of $B_s^0 \rightarrow J/\Psi f_0$, parameter r varies from 0 to 6 and δ from 0 to 2π

$$9.1 \times 10^{-5} < Br(B_s \rightarrow J/\psi f_0(980)) < 9.6 \times 10^{-5}, \quad (17)$$

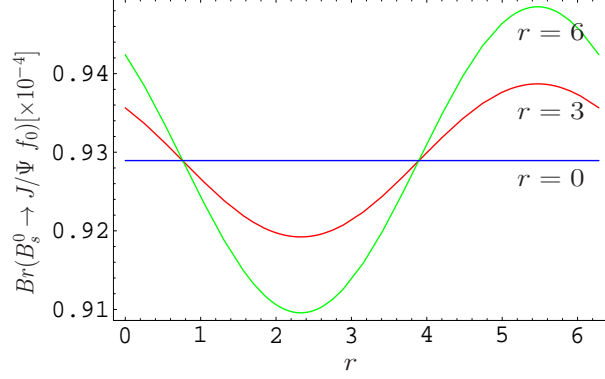


FIG. 3: The variation of the the branching ratios of $B_s^0 \rightarrow J/\Psi f_0$ with phase δ , three curves are for parameter $r = 0, 3, 6$, respectively.

which is shown from Fig. (2),(3).

With the value of the branching ratio of $f_0(980) \rightarrow \pi^+\pi^-$ in Ref. [32],

$$Br(f_0(980) \rightarrow \pi^+\pi^-) = 0.50_{-0.09}^{+0.07} \quad (18)$$

we can get the branching ratio of $B_s^0 \rightarrow J/\psi f_0(980); f_0(980) \rightarrow \pi^+\pi^-$,

$$4.55 \times 10^{-5} < Br(B_s^0 \rightarrow J/\Psi f_0; f_0(980) \rightarrow \pi^+\pi^-) < 4.8 \times 10^{-5} \quad (19)$$

This prediction of the branching ratio is about half of the averaged experimental data[9–11]

,

$$Br^{exp}(B_s^0 \rightarrow J/\Psi f_0; f_0(980) \rightarrow \pi^+\pi^-) = (1.20_{-0.21}^{+0.25}(\text{stat.})_{-0.19}^{+0.17}(\text{syst.})) \times 10^{-4} \quad (20)$$

So, it seems that the result from QCDF can not accomodate the experimental data. The reason is that the divergent integral in hard spectator correction is approximately expressed by the parameters, which are suitable for the modes in which hard spectator correction has little contribution. .

C. Hard Spectator Scattering Corrections In pQCD Approach

The divergence in the hard spectator correction arises from the neglect of transverse momentum. Using pQCD approach can avoid the divergence in the calculation of the hard spectator scattering corrections because transverse momentum of quarks is kept. The characteristic hard scale in the hard spectator scattering corrections is higher than that in B_s

meson transition form factor [33]. Therefore, we can employ pQCD approach based on k_T factorization theorem, which is free from the end-point singularity for the spectator amplitude [28]. In pQCD approach, the nonfactorizable hard spectator amplitudes can be written as,

$$A_{HS} = V_c^* \mathcal{M}_1^{(J/\psi f_0)} - V_t^* \mathcal{M}_4^{(J/\psi f_0)} - V_t^* \mathcal{M}_6^{(J/\psi f_0)}, \quad (21)$$

where the amplitudes $\mathcal{M}_{1,4}^{(J/\psi f_0)}$ and $\mathcal{M}_6^{(J/\psi f_0)}$ come from the $(V - A)(V - A)$ and $(V - A)(V + A)$ operators in Eq. (2), respectively. Their factorization formulas are given by pQCD approach

$$\begin{aligned} \mathcal{M}_{1,4}^{(J/\psi f_0)} = & 8\pi m_{B_s}^4 C_F \sqrt{2N_c} \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \Phi_{B_s}(x_1, b_1) \\ & \times \left\{ \left[((r_2^2 - 1)(\psi^t(x_2)r_2^2 + 2\psi^L(x_2)(r_2^2 - 1)(x_1 + x_2 - 1))\phi_{f_0}(x_3) \right. \right. \\ & + 2\psi^L(x_2)r_3((x_1 - x_3)r_2^2 + x_3)\phi_{f_0}^s(x_3) \\ & + (x_3 - r_2^2(x_1 + 2x_2 + x_3 - 2))\phi_{f_0}^\sigma(x_3)) \Big] \\ & \times E_{1,4}(t_d^{(1)})h_d^{(1)}(x_1, x_2, x_3, b_1) \\ & - \left[(r_2^2 - 1)(\psi^t(x_2)r_2^2 - 2\psi^L(x_2)(x_2r_2^2 - x_3r_2^2 - x_1 + x_2 + x_3))\phi_{f_0}x_3 \right. \\ & - 2r_3(\psi^L(x_2)((x_1 - x_3)r_2^2 + x_3)\phi_{f_0}^s(x_3) \\ & + (2\psi^t(x_2)r_2^2 + \psi^L(x_2)(r_2^2(x_1 - 2x_2 + x_3) - x_3))\phi_{f_0}^\sigma(x_3)) \Big] \\ & \times E_{1,4}(t_d^{(2)})h_d^{(2)}(x_1, x_2, x_3, b_1), \end{aligned} \quad (22)$$

$$\begin{aligned} \mathcal{M}_6^{(J/\psi f_0)} = & 8\pi m_{B_s}^4 C_F \sqrt{2N_c} \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \Phi_{B_s}(x_1, b_1) \\ & \times \left\{ \left[(r_2^2 - 1)(\psi^t(x_2)r_2^2 + 2\psi^L(x_2)((x_2 + x_3 - 1)r_2^2 + x_1 + x_2 - x_3 - 1))\phi_{f_0}x_3 \right. \right. \\ & - 2r_3(\psi^L(x_2)((x_1 - x_3)r_2^2 + x_3)\phi_{f_0}^s(x_3) \\ & + (2\psi^t(x_2)r_2^2 + \psi^L(x_2)(r_2^2(x_1 + 2x_2 + x_3 - 2) - x_3))\phi_{f_0}^\sigma(x_3)) \Big] \\ & \times E_6(t_d^{(1)})h_d^{(1)}(x_1, x_2, x_3, b_1) \\ & - \left[(r_2^2 - 1)(\psi^t(x_2)r_2^2 + 2\psi^L(x_2)(r_2^2 - 1)(x_1 - x_2))\phi_{f_0}x_3 \right. \\ & + 2\psi^L(x_2)r_3((x_1 - x_3)r_2^2 + x_3)\phi_{f_0}^s(x_3) \\ & + (x_3 - r_2^2(x_1 - 2x_2 + x_3))\phi_{f_0}^\sigma(x_3) \Big] \\ & \times E_6(t_d^{(2)})h_d^{(2)}(x_1, x_2, x_3, b_1) \Big\}, \end{aligned} \quad (23)$$

with the color factor $C_F = 4/3$, the number of colors $N_c = 3$, the symbol $[dx] \equiv dx_1 dx_2 dx_3$ and the mass ratio $r_{f_0} = m_0^{f_0 \bar{s}s} / m_{B_s}$, $m_0^{f_0 \bar{s}s}$ being the chiral scale associated with the f_0 meson. In the calculation of $\mathcal{M}_{1,4}^{(J/\psi f_0)}$ and $\mathcal{M}_6^{(J/\psi f_0)}$, we reserve the power terms of r_2 up to $\mathcal{O}(r_2^4)$, the power terms of r_3 up to $\mathcal{O}(r_3^2)$, because J/ψ meson is heavy.

In the derivation of spectator correction in pQCD approach, we need to input the wave function of relevant mesons, we list the wave functions in appendix. The evolution factors are written as [28]

$$E_i(t) = \alpha_s(t) a'_i(t) S(t)|_{b_3=b_1}, \quad (24)$$

with the Wilson coefficients,

$$\begin{aligned} a'_1 &= \frac{C_2}{N_c}, \\ a'_4 &= \frac{1}{N_c} \left(C_4 + \frac{3}{2} e_c C_{10} \right), \\ a'_6 &= \frac{1}{N_c} \left(C_6 + \frac{3}{2} e_c C_8 \right). \end{aligned} \quad (25)$$

The Sudakov exponent is given by [28]

$$\begin{aligned} S(t) &= S_{B_s}(t) + S_{f_0}(t), \\ S_{B_s}(t) &= \exp \left[-s(x_1 P_1^+, b_1) - \frac{5}{3} \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) \right], \\ S_{f_0}(t) &= \exp \left[-s(x_3 P_3^-, b_3) - s((1-x_3)P_3^-, b_3) - 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) \right], \end{aligned} \quad (26)$$

The hard functions $h_d^{(j)}$, $j = 1$ and 2 , are

$$\begin{aligned} h_d^{(j)} &= [\theta(b_1 - b_2) K_0(DM_{B_s} b_1) I_0(DM_{B_s} b_2) \\ &\quad + \theta(b_2 - b_1) K_0(DM_{B_s} b_2) I_0(DM_{B_s} b_1)] \\ &\quad \times K_0(D_j M_{B_s} b_2), \quad \text{for } D_j^2 \geq 0, \\ &\quad \times \frac{i\pi}{2} H_0^{(1)}(\sqrt{|D_j^2|} M_{B_s} b_2), \quad \text{for } D_j^2 \leq 0, \end{aligned} \quad (27)$$

with the variables,

$$\begin{aligned} D &= x_1 x_3 - x_1 x_3 r_2^2 - r_3^2 x_3^2, \\ D_1 &= x_1 x_3 + x_2 x_3 - x_3 + (-x_2^2 - x_1 x_2 - x_3 x_2 + 2x_2 + x_1 - x_1 x_3 + x_3 - 1) r_2^2 \\ &\quad + r_3^2 (-x_3^2 - x_2 x_3 + x_3) + \frac{1}{4} r_2^2, \\ D_2 &= x_1 x_3 - x_2 x_3 + (-x_2^2 + x_1 x_2 + x_3 x_2 - x_1 x_3) r_2^2 + r_3^2 (x_2 x_3 - x_3^2) + \frac{1}{4} r_2^2. \end{aligned} \quad (28)$$

In the calculation of hard function, Considering the heavy J/ψ meson, we reserve the power terms of r_2 up to $\mathcal{O}(r_2^4)$, the power terms of r_3 up to $\mathcal{O}(r_3^2)$.

The hard scales t are chosen as

$$t^{(j)} = \max(\sqrt{D}m_{B_s}, \sqrt{|D_j|m_{B_s}}, 1/b_1) . \quad (29)$$

D. Numerical Analysis

From the Eq. (8) and Eq. (9), we can derive the relation of the branching ratio of $B_s \rightarrow J/\psi f_0(\sigma)$ with $f_0 - \sigma$ mixing angle θ ,

$$Br(B_s^0 \rightarrow J/\Psi f_0) = \frac{1}{32\pi M_{B_s} \Gamma_{B_s^0}} G_F^2 (1 - r_2^2 + \frac{1}{2}r_2^4 - r_{3(f_0)}^2) \cos^2 \theta |A_{FA} + A_{VERT} + A_{HS}|^2 \quad (30)$$

$$Br(B_s^0 \rightarrow J/\Psi \sigma) = \frac{1}{32\pi M_{B_s} \Gamma_{B_s^0}} G_F^2 (1 - r_2^2 + \frac{1}{2}r_2^4 - r_{3(\sigma)}^2) \sin^2 \theta |A_{FA} + A_{VERT} + A_{HS}|^2 \quad (31)$$

where $r_{3(f_0)} = m_{f_0}/m_{B_s}$, $r_{3(\sigma)} = m_\sigma/m_{B_s}$, $\Gamma_{B_s^0}$ is the total decay width of B_s^0 meson.

To calculate the the branching ratio of $B_s^0 \rightarrow J/\Psi f_0$, it is necessary to take some parameters and distribution amplitude for relevant mesons as inputs.

The parameters and distribution amplitude for the relevant mesons used in this paper are listed in the Appendix.

The $f_0 - \sigma$ mixing angle θ lies in the ranges $25^\circ < \theta < 40^\circ$ and $140^\circ < \theta < 165^\circ$ [22]. According to Eq. (30), we can get the branching ratio of $B_s^0 \rightarrow J/\psi f_0$,

$$Br(B_s^0 \rightarrow J/\psi f_0) = (2.43_{-0.31}^{+0.30}(\omega_{B_s})) \times 10^{-4}, \quad (32)$$

The main theoretical error of $Br(B_s^0 \rightarrow J/\psi f_0)$ is induced by the uncertainty of shape factor ω_{B_s} of B_s meson wave function in Eq. (42).

With the value of the branching ratio of $f_0(980) \rightarrow \pi^+\pi^-$ in in Eq. (18), we can get the branching ratio of $B_s^0 \rightarrow J/\psi f_0(980); f_0(980) \rightarrow \pi^+\pi^-$,

$$Br(B_s^0 \rightarrow J/\Psi f_0; f_0(980) \rightarrow \pi^+\pi^-) = (1.215_{-0.155}^{+0.15}(\omega_{B_s})_{-0.21}^{+0.17}(f_0)) \times 10^{-4} \quad (33)$$

The first theoretical error of $Br(B_s^0 \rightarrow J/\Psi f_0; f_0(980) \rightarrow \pi^+\pi^-)$ is from the uncertainty of shape factor ω_{B_s} of B_s meson wave function, the second one is induced by the uncertainty of the branching ratio of $f_0(980) \rightarrow \pi^+\pi^-$ in Eq. (18).

Compared with the averaged experimental data[9–11] ,

$$Br^{exp}(B_s^0 \rightarrow J/\Psi f_0; f_0(980) \rightarrow \pi^+\pi^-) = (1.20_{-0.21}^{+0.25}(\text{stat.})_{-0.19}^{+0.17}(\text{syst.})) \times 10^{-4} \quad (34)$$

our prediction is in consistency with the experimental value.

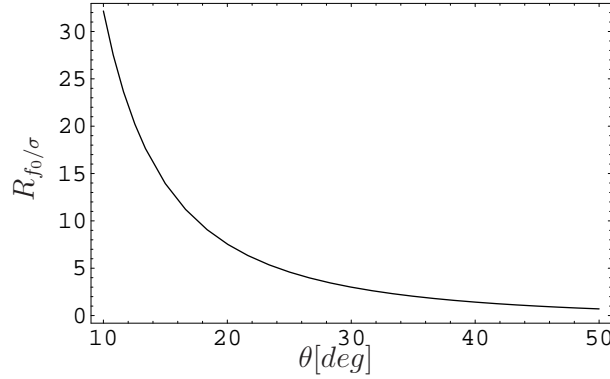


FIG. 4: The ratio of the branching ratios of $B_s^0 \rightarrow J/\Psi f_0$ to that of $B_s^0 \rightarrow J/\Psi \sigma$ as function of $f_0 - \sigma$ mixing angle θ

There are two ways to extract the $f_0 - \sigma$ mixing angle θ . One is based on the relation of the branching ratio of $B_s \rightarrow J/\psi f_0(980)$ with mixing angle θ shown in Eq.(30). We can determine $f_0 - \sigma$ mixing angle θ .

$$\theta = (34.03_{-10.5}^{+5.1}(\text{exp})_{-9.1}^{+5.1}(f_0)_{-6.4}^{+4.8}(\omega_{B_s}))^\circ \quad (35)$$

or

$$\theta = (145.97_{-5.1}^{+10.5}(\text{exp})_{-5.1}^{+9.1}(f_0)_{-4.8}^{+6.4}(\omega_{B_s}))^\circ \quad (36)$$

The first error is from experimental error of the branching ratio of $B_s^0 \rightarrow J/\psi f_0(980)$, the second one is due to the error of the branching ratio of $f_0(980) \rightarrow \pi^+\pi^-$, the third one is induced by the uncertainty of shape factor ω_{B_s} of B_s meson wave function. There are also other theoretical errors in our calculations, such as the uncertainty of final state meson wave functions and the known higher order corrections. Needless to see, the uncertainty of obtained measurement through this method is large.

In this direction, we can also use the experimental date of $B_s \rightarrow J/\psi f_0(980)$ and Eq. (30,31) to predict the branching ratio of $B_s \rightarrow J/\psi \sigma$,

$$Br(B_s^0 \rightarrow J/\psi \sigma) = (4.72_{-0.62}^{+0.66}(f_0)_{-0.59}^{+0.62}(\omega_{B_s})) \times 10^{-5}. \quad (37)$$

To determine phase β_s in B_s^0 mixing accurately for probe of NP, it is necessary to determine $f_0 - \sigma$ mixing angle more accurately. We need take the second method to determine the $f_0 - \sigma$ mixing angle θ with less uncertainties. From Eq. (30,31), we can get the relation of the ratio of the branching ratios of $B_s^0 \rightarrow J/\Psi f_0$ and $B_s^0 \rightarrow J/\Psi \sigma$ with the $f_0 - \sigma$ mixing angle θ ,

$$R_{f_0/\sigma} = \frac{Br(B_s^0 \rightarrow J/\Psi f_0)}{Br(B_s^0 \rightarrow J/\Psi \sigma)} = \cot^2 \theta \frac{(1 - r_2^2 + \frac{1}{2}r_2^4 - r_{3(f_0)}^2)}{(1 - r_2^2 + \frac{1}{2}r_2^4 - r_{3(\sigma)}^2)}, \quad (38)$$

The mass of $f_0(\sigma)$ is far less than that of B_s meson, so $r_{3(f_0(\sigma))}^2$ is negligible.

The Eq. (38) can be reduced into,

$$R_{f_0/\sigma} = \frac{Br(B_s^0 \rightarrow J/\Psi f_0)}{Br(B_s^0 \rightarrow J/\Psi \sigma)} = \cot^2 \theta, \quad (39)$$

This means that the mixing angle θ can be extracted from the ratio of the branching ratios of $B_s^0 \rightarrow J/\psi f_0(\sigma)$ with negligible theoretical uncertainty. The uncertainty of θ determined in this method is mainly from the uncertainty of the measured ratio of the branching ratios of $B_s^0 \rightarrow J/\psi f_0(\sigma)$. In Fig.4 we show the variation of the ratio of the branching ratios of $B_s^0 \rightarrow J/\psi f_0(\sigma)$ with θ . If the ratio of the branching ratios of $B_s^0 \rightarrow J/\psi f_0$ to that of $B_s^0 \rightarrow J/\psi \sigma$ were measured, we could determine the mixing angle θ fairly well. This is a good news to determine phase β_s in B_s^0 mixing accurately for the probe of NP.

III. SUMMARY AND DISCUSSION

In this paper, we derive the decay amplitude of $B_s^0 \rightarrow J/\psi f_0(\sigma)$ and the relation of the branching ratios of $B_s^0 \rightarrow J/\psi f_0(\sigma)$. We computed the factorizable contributions in QCDF approach and the hard spectator scattering diagrams in the perturbative QCD approach. The branching ratio of $B_s^0 \rightarrow J/\psi f_0$ is in agreement with recent experimental data. We also predict the branching ratio of $B_s^0 \rightarrow J/\psi \sigma$ to be $(4.72_{-0.62}^{+0.66}(f_0)_{-0.59}^{+0.62}(\omega_{B_s})) \times 10^{-5}$. We suggest two methods to determine the mixing angle θ of f_0 and σ . For the first method we get $f_0 - \sigma$ mixing angle θ to be about $(34.03_{-10.5}^{+5.1}(exp)_{-9.1}^{+5.1}(f_0)_{-6.4}^{+4.8}(\omega_{B_s}))^\circ$

or $(145.97_{-5.1}^{+10.5}(\exp)_{-5.1}^{+9.1}(f_0)_{-4.8}^{+6.4}(\omega_{B_s}))^\circ$, which is in consistency with others. The second method for determining the mixing angle θ has little theoretical uncertainty, but needs the experimental data of both the branching ratio of $B_s^0 \rightarrow J/\psi f_0(\sigma)$ as an input. We hope that the future experiment will measure it.

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Appendix: Input Parameters And Wave Functions

We use the following input parameters in the numerical calculations[1, 13, 34]

$$\begin{aligned} \Lambda_{\overline{\text{MS}}}^{(f=4)} &= 250\text{MeV}, & f_{B_s} &= (0.231 \pm 0.015)\text{GeV}, & M_{B_s} &= 5.366\text{GeV}, \\ M_W &= 80.41\text{GeV}, & \tau_{B_s^0} &= 1.472 \times 10^{-12}\text{s}, \end{aligned} \quad (40)$$

For the CKM matrix elements, we adopt the wolfenstein parametrization for the CKM matrix up to $\mathcal{O}(\lambda^3)[1]$,

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (41)$$

with the parameters $\lambda = 0.2253$, $A = 0.808$, $\bar{\rho} = 0.132$ and $\bar{\eta} = 0.341$.

For the B_s meson distribution amplitude, we adopt the model[35]

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp \left[-\frac{M_{B_s}^2 x^2}{2\omega_{B_s}^2} - \frac{1}{2}(\omega_{B_s} b)^2 \right], \quad (42)$$

where ω_{B_s} is a free parameter and we take $\omega_{B_s} = 0.5 \pm 0.05$ GeV in numerical calculations, and $N_{B_s} = 63.6708$ GeV is the normalization factor for $\omega_{B_s} = 0.5$ GeV.

The J/ψ meson asymptotic distribution amplitudes are given by [36]

$$\Psi^L(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x(1-x) \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7},$$

$$\Psi^t(x) = 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_c}} (1-2x)^2 \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \quad (43)$$

The wave function for $s\bar{s}$ components of $f_0(\sigma)$ meson are given as in Ref. [22]

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